Median in spatial and frequency domain filtering

"Give me a lever long enough, and a prop strong enough, and I can singlehandedly move the world."

—**Archimedes**

Figure 1. Impulsive noise filtering using a preliminary exponential noise detector. (a) The input image is corrupted by impulsive noise (15% corruption rate); (b) the result of filtering using the exponential noise detector; (c) the result of filtering using the simple median filter.

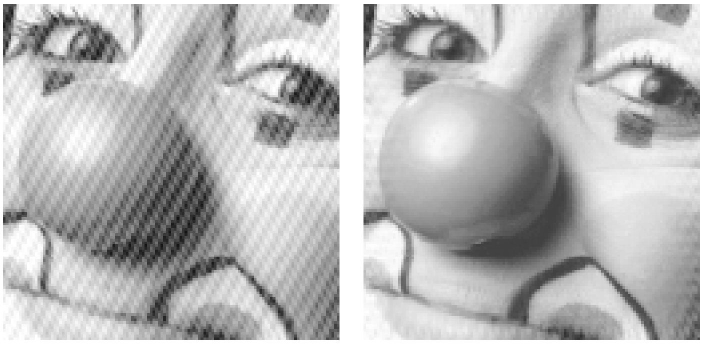

Figure 2. Quasi-periodic noise removal using the median detector and Gaussian surface filtering in the frequency domain. (a) The input noisy image. (b) The filtering result.

We would like to discuss the development of the notion of the median, which is widely used for impulsive-noise filtering. This notion introduces an important criterion for impulse detection in its local window (we will call the image values taken from the filtering window and sorted in ascending order as a *variational set*). Normally, the removal of impulses in an image by replacing their value by the median would be considered a very bad idea, because it is a destructive measure. But the median filter is based on the idea that the impulse will always lie on one of the ends of the variational set. This gives an excellent criterion for impulse detection, which can be done *a priori* to any filtering. To be more specific, the only thing needed is to check the observed pixel value, to see if it lies close to one of the ends of variational set taken from the pixel's local window. The closer it is to one of the ends, the higher the chance that it contains an impulse that has to be eliminated. Depending on the distance, a different number of pixels will be filtered: but this is still much better than filtering of *all* the pixels. This simple impulse detector can be used not just in impulse noise filters, but also other filters so that impulses will not affect the result. The simplest example of such filter is probably the a-trimmed mean filter.

Back to the main subject: a few interesting points have been raised. The first is the detection the impulses in spatial domain, performed by checking the distance from the ends of the variational set. However, if we look at it from the opposite point of view, this is the same as checking the distance from the median. It turns out that by applying an exponential function to the difference between the observed pixel value and the median leads to even stronger impulse detection. This method can effectively detect impulsive noise on the image with a corruption rate of 10% (see Figure 1). Here, we compare the exponent of difference to some threshold: if it's higher we say that pixel is corrupted. The steepness of the exponent can be modified to adjust the sensitivity to threshold.

Now let us jump to the frequency domain or, more precisely, the Fourier spectrum's amplitude domain. It is well known that periodic and quasi-periodic noise is represented by peaks in the Fourier spectrum. This suggests the idea of using the median for removal of these peaks. Here, the coefficient of interest is not checked for its distance from the ends of variational set (which is built of spectral coefficients around the analyzed coefficient), nor for distance from the median. Instead, the ratio of the coefficient-of-interest to the median is calculated. This ratio is compared to the threshold and, if it's larger then the coefficient, it is considered to be a peak and eliminated. This is the idea behind the spectral peak detector.

From here, ideas for two peak eliminators are suggested. The first involves the replacement of the spectral coefficient by the median, just as in the usual median filter, flattening the peak and the whole surface around it. The second approach requires more explanation. The periodic distortions in the spectrum rarely take the form of a single impulse: they usually look like a steep hill. Using the median to replace the peak will smooth this hill somewhat, but not eliminate it. These hills look very similar to a two-dimensional Gaussian surface, which suggests the idea of taking this surface (its values must vary from 0 to 1), inverting it by subtracting it from 1 and multiplying the spectral hill by this surface. This way the hill will be completely removed and, a possible drawback of this method, the peak will be set to 0. Of course, a scaling coefficient can be introduced so the peak will not be set to 0 but just reduced by some amount. Also, the steepness of the Gaussian surface can be modified to better filter noise.

Consequently, these approaches work well for different scenarios: median is good when the periodic structure introduces singular peaks, or peaks with a few smaller peaks around it. The surface method is best when the periodic structure introduces wide hills (see Figure 2). Of course, it should be noted that the surface could be compressed such that it will just replace the peak (one coefficient) with 0 and leave the vicinity intact. But, for this scenario, the median technique is preferred. The third and final modification to the surface technique is to adapt it to each filtering window such that, after filtering the peak, it will become equal to the median instead of 0. This necessitates the scaling of the surface.

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